Statistics

Summer 2023

Lecture 13

Feb 19-8:47 AM

Class QZ 16

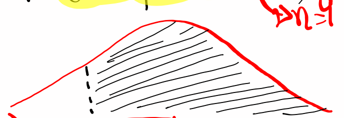
Given $N(150, 18)$ → Normal dist. → $\sigma = 18$ → $\mu = 150$

For randomly selected groups of 9 → $n = 9$

1) Find $P(\bar{x} > 140)$

$= \text{normalcdf}(140, E99, 150, 6)$

$= \boxed{.952}$ ✓



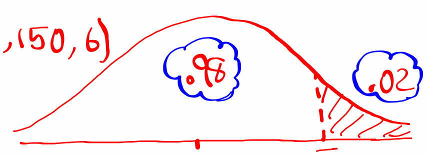
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 150 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{9}} = 6 \end{cases}$

2) Find $\bar{x} = P_{98}$, Round to 1-decimal.

$\bar{x} = P_{98} = \text{invNorm}(.98, 150, 6)$

$= 162.322$

$\approx \boxed{162.3}$ ✓



$\mu_{\bar{x}} = \mu = 150$ $\quad \bar{x}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{9}} = 6$

Jul 3-11:29 AM

SG22

$\alpha \rightarrow$ Alpha

$0 < \alpha < 1$

$\alpha \rightarrow$ Significance level

$1-\alpha \rightarrow$ Confidence level $\rightarrow$ %

$\alpha = .02$

$1-\alpha = .98 \rightarrow$ Conf. level $= 98\%$

$\alpha = .1$

$1-\alpha = .9 \rightarrow$ C-level $= 90\%$

Suppose C-level $= 99\% = .99$

$1-\alpha = .99 \rightarrow \alpha = .01$ Sign. level

If α not given $\Rightarrow$ use .05

If C-level not given $\Rightarrow$ use 95%

Jul 5-7:37 AM

$(1-\alpha)\cdot 100\% \Rightarrow$ Conf. level (C-level)

Conf. level is the middle area of the graph of prob. dist. Curve.

90% C-level $\Rightarrow$ Middle area .9

$1 - .9 = .1 \leftarrow \alpha$

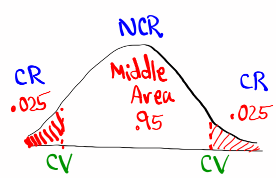
$.1 \div 2 = .05 \leftarrow \alpha/2$

C-level: $95\% = .95$

Middle area $= .95$

$1 - .95 = .05 \leftarrow \alpha$

$.05 \div 2 = .025 \leftarrow \alpha/2$



$1-\alpha$ Region $\rightarrow$ Non-Critical Region $\rightarrow$ NCR

α Region $\rightarrow$ Critical Region $\rightarrow$ CR

$\alpha/2$ area on each tail

Value/values that Separate CR and NCR are Called Critical Values (CV)

Jul 5-7:42 AM

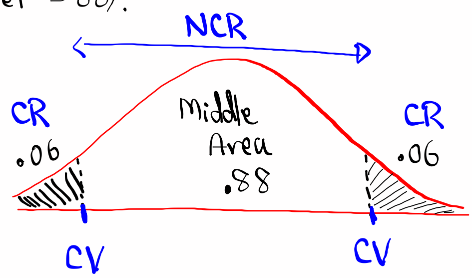
Suppose Conf. level = 88%.

C-level: .88

$1-\alpha = .88$

$\boxed{\alpha = .12}$

$\boxed{\alpha/2 = .06}$



Suppose $\alpha = .03$

$\alpha/2 = .03/2 = .015$

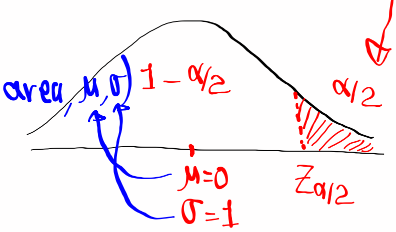
$1-\alpha = 1-.03 = .97$

Do completing drawing with bell-shape and all labels.

$Z_{\alpha/2}$ is the Critical Value, Round to 3-decimals, that separates the right-area of $\alpha/2$ from the rest in Standard normal Prob. dist. Curve.

How to find $Z_{\alpha/2}$

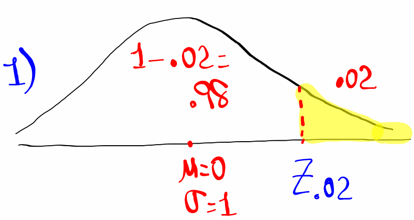
Use invNorm(Left area, μ, σ)



Find $Z_{.02}$

$Z_{.02} = \text{invNorm}(.98, 0, 1)$

$= \boxed{2.054}$

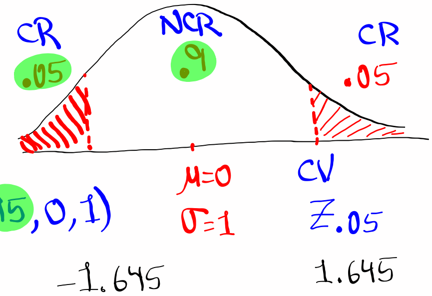


Find $z_{\alpha/2}$ for $\underbrace{90\%\text{ C-level.}}_{\text{Middle Area} = .9}$

$1-\alpha = .9$

$\alpha = .1$

$\alpha/2 = .05$



$z_{.05} = \text{invNorm}(.95, 0, 1)$

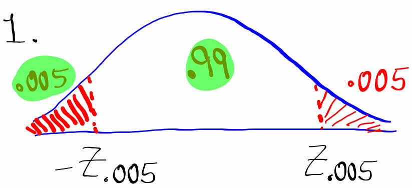
$= \boxed{1.645}$

Find $z_{\alpha/2}$ for $\alpha = .01$.

$\alpha/2 = .005$

$1-\alpha = .99$

C-level: $99\% = .99$



$z_{.005} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Jul 5-8:01 AM

Evaluate $1.960 \cdot \sqrt{\frac{(.4)(.6)}{25}}$, Round to 2-decimals.

$\approx \boxed{.19}$

Suppose $z_{\alpha/2} = 2.576$, $p = .8$, $n = 36$

Evaluate $z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}}$

$q = 1 - p$
$= .2$

Round to 2-decimals.

$2.576 \cdot \sqrt{\frac{(.8)(.2)}{36}} = \boxed{.17}$

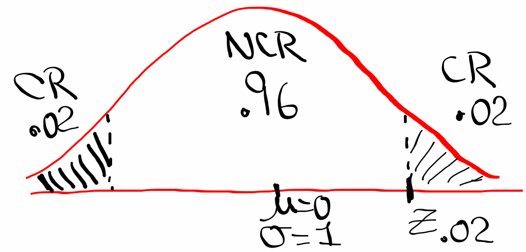
Jul 5-8:10 AM

Find $\boxed{z_{\alpha/2} \text{ for } 96\% \text{ C-level}}$ then evaluate

$z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}}$ for $n=16$ and $p=.5$.

$q = 1 - p = .5$

Round to 2-decimal places.



$z_{\alpha/2} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

Now

$2.054 \cdot \sqrt{\frac{(.5)(.5)}{16}}$

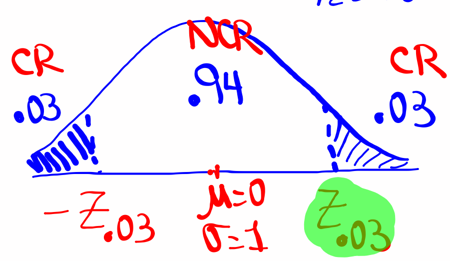
$\approx \boxed{.26}$

Jul 5-8:15 AM

Evaluate $z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}}$, Round to 2-decimal places, given $p=.75$, $n=100$, and $\alpha=.06$.

$\alpha/2 = .03$

$q = 1 - p = .25$



$1.881 \cdot \sqrt{\frac{(.75)(.25)}{100}}$

$z_{.03} = \text{invNorm}(.97, 0, 1) = \boxed{1.881}$

$\approx \boxed{.08}$

Jul 5-8:19 AM

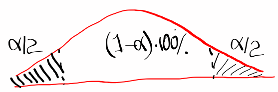
Population ⟷ Parameter

Sample ⟷ Statistic

we use Statistic to estimate Parameter.

Every estimation comes with Some level of Confidence.

level of Conf. ⟹ Confidence level $(1-\alpha)\cdot 100\%$



where $0<\alpha<1$

α Significance level

Conf. level is the middle area.

$\alpha/2$ is the area on each Side.

If α not given, $\boxed{\text{use } .05}$

If C-level not given, $\boxed{\text{use } 95\%}$

Jul 5-8:41 AM

If we wish to estimate	use
Population Proportion p	Sample Proportion $\boxed{\hat{p}}$ p-hat
Population mean μ	Sample Mean $\boxed{\bar{x}}$ x-bar
Population Standard deviation σ	Sample Stand. Dev. $\boxed{s}$
Parameters	Point-estimate "Statistic"

we start with point-estimate and Some level of Confidence to find range of values for Corresponding Parameter.

Confidence Interval

Conf. Interval is a range of values that the parameter falls within with a desired Conf. level.

Jul 5-8:47 AM

Estimating Population Parameter P:

$$<P<$$

$$\hat{p} - E < P < \hat{p} + E$$

$\hat{p}$ Sample proportion $\quad \hat{q} = 1 - \hat{p}$

$\hat{p} = \dfrac{x}{n}$ ← # of Favorable Responces (x); ← Sample Size (n)

E Margin of Error

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$Z_{\alpha/2}$ is the Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

Jul 5-8:56 AM

I Surveyed 100 Students and 80 of them had iPhone.

$n = 100 \qquad \hat{p} = \dfrac{x}{n} = \dfrac{80}{100} = .8$

$x = 80 \qquad \hat{q} = 1 - \hat{p} = .2$

C-level: .9

Find 90% Conf. interval for the prop. of all Students that have iPhone.

$E = Z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}\hat{q}}{n}} \qquad <P<$

$\hat{p} - E < P < \hat{p} + E$

$= 1.645 \cdot \sqrt{\dfrac{(.8)(.2)}{100}} = \boxed{.07} \qquad .8 - .07 < P < .8 + .07$

$\boxed{.73 < P < .87}$ ✓

$Z_{.05} = \text{invNorm}(.95, 0, 1) =$

we are 90% Confident that between 73% & 87% of all Students have iPhone.

Using TI

$\boxed{\text{STAT}}$ → → TESTS ↓↓ $\boxed{\text{1-PropZInt}}$

$x = 80$

$n = 100$

C-level: .9

$\boxed{\text{Calculate}}$

$.734 < P < .866$

$\boxed{.73 < P < .87}$

$\hat{p} = \dfrac{.87 + .73}{2} = \boxed{.8}$

$E = \dfrac{.87 - .73}{2} = \boxed{.07}$

Jul 5-9:00 AM

I surveyed 150 students, and 60% of them had a job while going to school.

$n = 150$

$\hat{p} = .6$

$\Rightarrow x = n\hat{p} = 150(.6) = 90$

if decimal $\Rightarrow$ Round-up

Find 98% Conf. interval for the prop. of all students that are working while going to School.

$\Rightarrow$ C-level: .98

$\boxed{.51 < P < .69}$

$\boxed{\text{1-PropZInt}}$

$x: 90$

$n = 150$

C-level: .98

$\boxed{\text{Calculate}}$

We are 98% confident that between 51% & 69% of all students are working while going to School.

$E = \frac{.69 - .51}{2} = \boxed{.09}$

$\hat{p} = \frac{.69 + .51}{2} = \boxed{.6}$

Jul 5-9:15 AM

In a survey of 180 students, 47.5% of them were in favor of online classes.

$n = 180$

$\hat{p} = .475$

$\Rightarrow x = n\hat{p} = 180(.475) = 85.5 \Rightarrow \boxed{x = 86}$

if decimal $\Rightarrow$ Round-up

Find Conf. interval for the prop. of all students in favor of online classes.

$\Rightarrow$ NO C-level $\Rightarrow$ use .95

$\boxed{.40 < P < .55}$

$\boxed{\text{1-PropZInt}}$

$x = 86$

$n = 180$

C-level: .95

$\boxed{\text{Calculate}}$

We are 95% confident that between 40% and 55% of all students are in favor of online classes.

$E = \frac{.55 - .40}{2} = \boxed{.075}$

7.5%

$\hat{p} = \frac{.55 + .40}{2} = \boxed{.475}$

Jul 5-9:26 AM

Given: $.125 < P < .375$

1) Find $\hat{P} = \dfrac{.375 + .125}{2} = \boxed{.25}$ 25%

2) Find $E = \dfrac{.375 - .125}{2} = \boxed{.125}$ 12.5%

Calculate the following, $\boxed{\text{Round-up}}$ to a whole #

$$.6 \cdot .4 \left(\frac{2.576}{.05}\right)^2 = 637.034 \approx \boxed{638}$$

$$.25 \left(\frac{1.645}{.075}\right)^2 = 120.268 \approx \boxed{121}$$

Jul 5-9:35 AM

Estimate Population Mean μ:

$$< \mu <$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x} \rightarrow$ Sample Mean , $E \rightarrow$ Margin of error

Case I: σ Known	Case II: σ Unknown
$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	
Z Interval inpt: $\boxed{\text{Stat}}$	

Jul 5-10:03 AM

Given $n = 25$, $\bar{x} = 140$, $\sigma = 15$

C-level: .99 → middle area = .99

Find conf. interval for μ.

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 2.576 \cdot \frac{15}{\sqrt{25}} \approx \boxed{8}$ (7.728)

$\bar{x} - E < \mu < \bar{x} + E$

$140 - 8 < \mu < 140 + 8$

$\boxed{132 < \mu < 148}$

We are 99% confident that pop. mean falls within 132 & 148.

$Z_{.005} = \text{invNorm}(.995, 0, 1)$

Now TI

STAT TESTS ZInterval

Inpt: Stats

$\sigma = 15$

$\bar{x} = 140$

$n = 25$

C-level: .99

Calculate

$132.27 < \mu < 147.73$

$\boxed{132 < \mu < 148}$

because $\bar{x}$ was a whole #.

$E = \frac{148 - 132}{2} = \boxed{8}$

$\bar{x} = \frac{148 + 132}{2} = \boxed{140}$

Jul 5-10:07 AM

I randomly selected 25 students, and their mean age was 32.5 yrs.

$n = 25$ $\bar{x} = 32.5$

It is known that standard deviation of ages of all students is 8.6 yrs.

$\sigma = 8.6$

Find 90% conf. interval for the mean age of all students.

→ C-level: .9

$\boxed{29.7 < \mu < 35.3}$

σ known → ZInterval

Inpt: Stats

$\sigma = 8.6$

$\bar{x} = 32.5$

$n = 25$

C-level: .9

Calculate

Round to 1-decimal

Since $\bar{x}$ is in 1-decimal

$E = \frac{35.3 - 29.7}{2} = \boxed{2.8}$

$\bar{x} = \frac{35.3 + 29.7}{2} = \boxed{32.5}$

Jul 5-10:17 AM

I randomly Selected 18 exams, and here are the Scores:

85	70	92	100	80
75	80	68	95	90
55	100	80	70	65
78	98	90		

Find
1) $\bar{x} \approx 82$
2) $S \approx 13$
} Round to whole #

It is known that Standard deviation of Scores of all exams is 10. $\sigma = 10$

Find Conf. interval for the mean Score of all exams. → NO C-level ⇒ use .95

$\boxed{77 < \mu < 87}$

Since σ Known (Given) → use Z Interval $\boxed{\text{Stats}}$

Since $\bar{x}$ is whole #

Inpt:
$\sigma = 10$
$\bar{x} = 82$
$n = 18$
C-level: .95
$\boxed{\text{Calculate}}$

$E = \frac{87 - 77}{2} = \boxed{5}$

$\bar{x} = \frac{87 + 77}{2} = \boxed{82}$

We are 95% Confident that the mean Score of all exams fall within 87 & 77.

2.5%	95%	2.5%
	77 — 87	

Jul 5-10:26 AM

Estimate Population Mean μ:

$$< \mu <$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x}$ → Sample Mean , E → Margin of error

Case I: σ Known	Case II: σ Unknown
$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$ → $df = n-1$
Z Interval inpt: $\boxed{\text{Stats}}$	T Interval inpt: $\boxed{\text{STATS}}$

Jul 5-10:03 AM

Given $n=12$, $\bar{x}=34$, $S=10$, C-level: .98

Find Confidence Interval for μ.

Since σ is unknown ⇒ T Interval

inpt: Stats

Since $\bar{x}$ is a whole #,

$\boxed{26 < \mu < 42}$

$E = \frac{42-26}{2} = \boxed{8}$

$\bar{x} = \frac{42+26}{2} = \boxed{34}$

If we were doing formula, $df = n-1 = \boxed{11}$

Jul 5-11:04 AM

I randomly selected 20 nurses, their mean monthly salary was \$6400 with standard deviation of \$500.

$n=20$, $\bar{x}=6400$, $S=500$

Find conf. interval for the mean monthly salary for all nurses.

→ No C-level → use .95

$\boxed{6166 < \mu < 6634}$

σ unknown → T Interval

Inpt: Stats ← whole #

$\bar{x}=6400$

$S=500$

$n=20$ ← $df = 20-1 = \boxed{19}$

C-level: .95

Calculate

$E = \frac{6634-6166}{2} = \boxed{234}$

$\bar{x} = \frac{6634+6166}{2} = \boxed{6400}$

Jul 5-11:08 AM

I randomly selected 10 exams, here are the scores:

84 96 100 80 70

75 85 68 72 55

Find

1) $\bar{x} = 79$

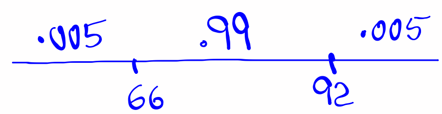
2) $S = 13$

Round to whole #

Find 99% conf. interval for the mean of all exams.

$\boxed{66 < \mu < 92}$

σ unknown $\rightarrow$ TInterval



$$E = \frac{92 - 66}{2} = \boxed{13} \qquad \bar{x} = \frac{92 + 66}{2} = \boxed{79}$$

Jul 5-11:16 AM